Simulation of Mathematical Model for Lung and Mechanical Ventilation

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Abstract

A mathematical model (MM) of artificial ventilation takes more positive evolution whether to represent the lung status during normal breathing or to represent the artificial ventilation. This paper presents a MM of Volume Controlled Ventilator (VCV) output signals, including Positive End Expiration Pressure (PEEP) and Dynamic Compliance (C). The proposed MM is expressed by linear, quadratic and exponential equations to represent the obtained combination of inspiration and expiration activities by VCV for ideal and practical lung cases. The MM of ventilator output signals is combined with the existed MM of lungs to represent the artificial ventilation process using VCV. The combined MMs are modelled and simulated using the Simulink tools in MATLAB program. The input (pressure) signal from VCV and the derived output (volume and flow) signals are monitored graphically using a constructed simulator. The results clarified the efficiency of the proposed MM and simulator. Moreover, the simulator has the ability to display instantaneous and continues waveforms of pressure, flow, volume and P-V loop similar to the real artificial ventilation. It also has the ability to be used as a good training tool for students.

Keywords: Volume controlled ventilator, Mathematical model, Lung model.

1. INTRODUCTION

Mechanical ventilator is an important therapeutic machine that supports breathing during patients' treating. There are a two-type volume controlled ventilator (VCV) and a pressure controlled ventilator (PCV). Both types deliver and control flow, pressure and volume of air and medical gases to patients' lungs [1]. Mathematical models (MMs) represent the supporting process by ventilators and the respiratory system. Thus, MMs, on the one hand, are beneficial for studying the respiratory system behavior, whether in normal or abnormal cases using VCV or PCV [2, 3, and 4]. On the other hand, MMs of artificial ventilations are important to monitor and analyze the ventilator performance while patients are being treated [5].

VCV or volume controlled mode takes an important concern in simulation and modeling because it is the most commonly used for adult and pediatric ventilations.

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Moreover, it companions other parameters which have effects on the mechanical ventilation control such as positive end expiration pressure (PEEP) [6, 7].

In this paper, the MM of VCV output signals are represented using linear, quadratic and exponential equations that combine with an already MM of healthy lungs. In addition, the proposed MM represents PEEP, as an important controlled parameter during the artificial ventilation, in order to reflect the efficiency of the MM respiratory activities.

The proposed MM is constructed and simulated in Simulink package in MATLAB. The obtained simulator has the ability to display the VCV output signal (pressure) and the modulated output signals of ventilation (flow and volume) that are monitored as continues waveforms in reality during the mechanical ventilation support. Furthermore, the obtained simulator can display the dynamic compliance (C) that reflects patients' responses during the artificial ventilation process.

Using the proposed method, the simulator can represent the VCV output signals during the artificial ventilation for an ideal case (healthy lung) and a practical case (unhealthy lung).

The rest of this paper is organized into three sections and a conclusion. The first presents a brief review of related works in the same trend. The second describes the methodology of the proposed MM, whereas the third shows the obtained results by illustrating curves (e.g. volume and flow), followed by an interpretation and discussion. The conclusion summarizes the efficiency of the proposed MM and constructions of the simulator successes followed by suggestions for future work.

2. RELATED WORK

The MM and simulation of respiratory system, mechanical ventilators or all artificial ventilation processes have developed rapidly because of their great impact on health care and medical application improvements. Consequently, many works have developed simulators using software tools such as MATLAB [6][8]. In research [9] the VCV ventilation system has been represented as a pneumatic system, where the output (pressure) is considered as an open thermodynamic system, and its work can be considered as an isothermal process. Then the MM of VCV system is derived. While other researchers [2,10] have expressed the mathematical model of the artificial ventilation process as a multi compartments model. Moreover, they have focused on the small detail behaviors of the respiratory system anatomy, where lungs have been represented and modeled as a series of electrical resistances and capacitances.

On the one hand, the lungs' MM and the respiratory activities of the pressure signal have been represented using the quadratic and exponential equations [11]. On the other hand, other researchers have used a transform function of the airflow (output of ventilator) that changes as a function of supported pressure supplied lungs [10]. The Eq. (1) shows the over obtained transform function, where all parameters values have been considered for healthy lungs. Of course, this function can be used to calculate unhealthy lungs by changing the values of lung resistance and compliance.

\[
\frac{H(s)}{P(s)} = \frac{(s^2 + 420s)}{(s^2 + 620s + 4000)}
\]  
(1)
3. THE PROPOSED METHOD

The proposed method consists of three parts as shown in Fig. (1).

![Diagram showing the proposed method]

Figure (1): MMs proposed method

3.1 Modeling the VCV output signal with PEEP

The first part of the proposed method presents the simulation of pressure signal of the VCV with PEEP in MATLAB. The input matrixes represent the time period of one breathing cycle (5 seconds approximately), where the inspiration time lasts up to 2 seconds and the expiration time lasts up to 3 seconds. In addition, the completed breathing cycle is modeled for an ideal case signal that represents the VCV with the respiratory system without load on the lung (healthy case) and a practical case signal that represents the dealing VCV with the lung (unhealthy case).

3.1.1 MM of VCV output signals, an ideal case

The inspiration activity is represented as a linear equation, see Eq. (3), and the expiration activity is represented as an exponential equation, see Eq. (4). The breathing cycle combines the inspiration and expiration equations, see Eq. (5).

\[
\begin{align*}
    y_1 &= a_0 \times x_1 \quad 0 \leq x_1 \leq 2 \\
    y_2 &= a_1 \times e^{-2x_2} \quad 2 \leq x_2 \leq 5 \\
    y_3 &= a_2 + y_1 + y_2
\end{align*}
\]

Where

- \(a_0\) is the parameter that represents a slope of the rising pressure of the inspiration,
- \(a_1\) is the parameter that represents a constant for the beginning of the expiration,
- \(a_2\) is the parameter that represents the constant value of PEEP;
- \(x_1\) and \(x_2\) represent the inspiration and expiration durations respectively.

3.1.2 MM of VCV output signals, a practical case

Similarly, to the ideal case, the inspiration and expiration activities and their combination are expressed using the following equations respectively:

\[
\begin{align*}
    y_1 &= a_3 \times x_1 - a_1 \times x_1^2 \quad 0 \leq x_1 \leq 2 \\
    y_2 &= a_2 \times e^{-2x_2} \quad 2 \leq x_2 \leq 5 \\
    y_3 &= a_3 + y_1 + y_2
\end{align*}
\]
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Where

- $a_1$ is the parameter that describes the shape of the wave,
- $a_2$ is the parameter that represents a constant for the beginning of the expiration;
- $a_3$ is the parameter that represents the constant value of PEEP,

Other parameters (as mentioned above).

3.2 Lung mathematical model

The MM of healthy lungs is represented using the transfer function, seen in the Eq. (1), which has been suggested by Michael C. [10] as a multi compartment MM of lungs. This model is combined with the suggested VCV MM. In the same way, this model has been improved to be used with the MM of PCV and to present normal and abnormal lung cases [3]. Therefore, the combination of the two models obtains the MM of artificial ventilation using VCV in reality.

3.3 Displaying the VCV signal curves

The output signals have been derived and displayed using a constructed simulator. The waveforms or curves are the monitored tools used for illustrating the behavior of the signals' flow ($F$) and volume ($V$), the dynamic compliance ($\Delta V/\Delta P$) that is referred to as P-V loop, and the pressure signal.

3.4 Simulator construction

The MM of VCV output signals and the lung transfer function are constructed and simulated using Simulink tools in MATLAB. The simulation sequences of generating pressure signals, respiratory activities and other processes are shown in Fig. (2).

4. RESULTS AND DISCUSSION

The simulation respiratory activities of the inspiration and expiration for VCV input signals in ideal and practical cases are illustrated graphically in Figures (3a and 3b).
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Figure (2): Simulation sequences

Figure (3): Complete breathing cycle of VCV: (a) Ideal case (b) Practical case
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Figures (3a and 3b) show the breathing cycle reflecting the cases of breathing cycle beginnings after PEEP that has been included in the MM for both cases; ideal and practical. These figures clearly illustrate the period (5 sec.) of the complete breathing cycle, as expressed in the MM. Moreover, the dotted line in Figure. (3b) shows the ideal case of the breathing cycle or the lung without loads (healthy case) to observe the difference between the lung status and the behavior of MM with both cases. This result indicates that the VCV MM effectively represents pressure signals for the lung status under supporting.

Figures (4 and 5) show the curves of pressure, volume and flow in ideal and practical cases respectively. These results indicate that the simulator can provide the monitoring respiration waveforms of the instantaneous pressure, airflow and lung volume; similarly, to the monitoring waveforms in reality. Hence, the monitored variable curves, which are mimicked the reality, and the instantaneous waveforms offer a significant ability over the waveforms that have illustrated instantaneousness for one respiratory cycle of pressure, airflow and lung volume in a similar work [11].

![Figure (4): Pressure, volume and flow of VCV signals in an ideal case](image)

Fig. (6) shows the curve of dynamic compliance loop (p-v loop). This result adds an important ability to the proposed MM and the constructed simulator because this curve describes the relation between the volume and pressure which indicates the patient level response to the mechanical ventilation. This characteristic offers a significant ability over related works [11, 12].

The developed MM and simulator provide significant improvements in representing the pressure signal of VCV including PEEP for ideal and practical cases at time of supporting the respiratory system. Moreover, the simulator may give a clear view of monitored signals: pressure, volume and flow during the ventilation process.
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In general, the PEEP has not been represented in developed MMs of previous studies [9] [11], while it is represented in the developed MM of this work using an easier method; linear, quadratic and exponential equations. Furthermore, the simulator can monitor the compliance curve (P-V loop) which determines the patient status response to the artificial ventilation process.

Figure (6): Compliance Curve (P-V loop)

5. CONCLUSION AND FUTURE WORK

This work aimed to model and simulate VCV output signals (pressure, volume, flow) and the lung status behavior during the mechanical ventilation by VCV. In addition, it aimed to display VCV signals and p-v curves continuously.

The simulator, using a presented method of modeling, provides a monitor for the main artificial ventilation variables of VCV, and helps making an idea of the artificial ventilation process in ideal and practical cases. Moreover, it provides instantaneous and continuous waveforms monitoring the VCV output signals and compliance.

The simulator may also be used to analyze ventilator performance. Therefore, it can be used as a good tool to study, monitor and describe the lung status and the VCV during the artificial ventilation process in training laboratories.

The linear, quadric and exponential equations are used in a simple way to formulate MMs of the only main variables of VCV signals and represent PEEP. However, there are other vital variables which control the breathing process and mechanical ventilation. Thus, the presented MM and simulator could be improved to represent more variables of machines and patients during the artificial ventilation. They also need further investigation and validation to achieve the analyzing ventilator performance with different lung statuses.
6. References


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